

Mathematics Talent Reward Programme

Question Paper for Class IX

17th January, 2016

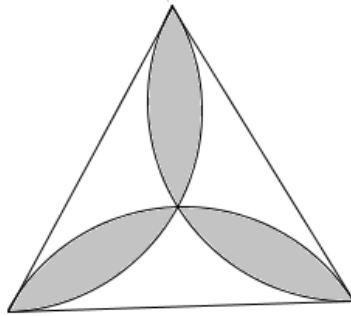
Total Marks: 150

Allotted Time: 2:00 p.m. to 4:30 p.m.

Multiple Choice Questions

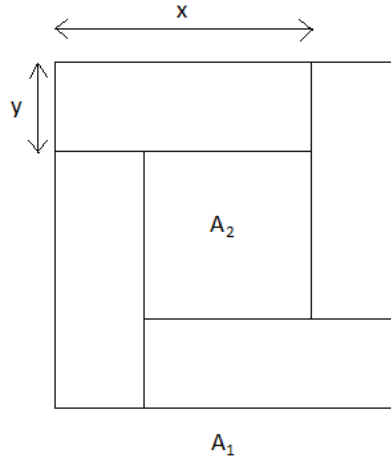
[You should answer these questions in the first page according to the order given in the question. Each question has only one correct option. You will be awarded 4 marks for the correct answer, 1 mark if the question is not attempted and 0 marks for wrong answer.]

- Radius of a cone is increased by 10% and height of the same cone is decreased by 10%, then the volume of the cone has increased by
(A) 8.3 %, (B) 8.6 %, (C) 8.9%, (D) 9.1%.
- Consider an equilateral triangle of length $\sqrt{6}$ as shown in the figure. Find the area of the shaded portion.



- (A) $\frac{3}{2}(2\pi - \sqrt{6})$, (B) $2\pi - 3\sqrt{3}$, (C) $\sqrt{6}\pi - \frac{1}{\sqrt{3}}$, (D) $\frac{3}{2}(3\pi - 2\sqrt{3})$.
- Perimeter of a triangle with sides a, b and c is 2. Then the expression $ab + bc + ca - abc - 1$ is
(A) always positive, (B) always negative, (C) 0, (D) None of these.
 - x, y are real numbers with $x + y = 1$ and $x^2 + y^2 = 2$. Then the value of $x^5 + y^5$ is given by
(A) $\frac{5}{2}$, (B) $\frac{17}{4}$, (C) $\frac{7}{2}$, (D) $\frac{19}{4}$.
 - A five digit number is called *Flappy* if product of its last two digits is 32 and sum of all five digits is 36. Suppose
$$x = \frac{\text{Number of Flappy numbers}}{\text{Number of Flappy numbers which are divisible by 36}}$$
Then x equals
(A) 1, (B) 3, (C) 2, (D) None of these.
 - You are given three bricks each measuring $5'' \times 4.5'' \times 3''$. How many different heights can you build up using all three of them?
(A) 14, (B) 7, (C) 10, (D) 13.
 - Let $x_1 = 2016$. For $n > 1$ define $x_n = \frac{n}{x_{n-1}}$. Then $x_1 x_2 \cdots x_{10} =$
(A) 2016, (B) 2280, (C) 3684, (D) None of these.
 - Vessel A has liquids X and Y in the ratio $X : Y = 8 : 7$. Vessel B holds a mixture of X and Y in the ratio $X : Y = 5 : 9$. What ratio should you mix the liquids in both vessels if you need the mixture to be $X : Y = 1 : 1$?
(A) 4:3, (B) 30:7, (C) 17:25, (D) 7:30.
 - How many six digit perfect squares can be formed using all the numbers 1,2,3,4,5,6 as digits?
(A) 5, (B) 19, (C) 7, (D) None of these.

10. 4 rectangles of same dimensions $x \times y$ are arranged in the following manner as shown in figure. Let A_1 be the area of the total square and A_2 be the area of the smaller square. Suppose $A_2 = \frac{1}{9}A_1$. Then $x : y$



- (A) 3 : 1, (B) 2 : 1, (C) 7 : 2, (D) 5 : 2.
11. Which of the following is true?
 (A) $2^{125} < 3^{75} < 5^{50}$, (B) $3^{75} < 2^{125} < 5^{50}$, (C) $5^{50} < 3^{75} < 2^{125}$, (D) $2^{125} < 5^{50} < 3^{75}$.
12. Let x be a positive real number. Then
 (A) $x^2 + \pi^2 + x^{2\pi} > x\pi + (x + \pi)x^\pi$, (B) $x^\pi + \pi^x > x^{2\pi} + \pi^{2x}$,
 (C) $x\pi + (x + \pi)x^\pi > x^2 + \pi^2 + x^{2\pi}$, (D) None of these.
13. Let $P(x) = (x - 1)^{21} + (x - 1)^{20}(1 - x) + (x - 1)^{19}(1 - x)^2 + \dots + (1 - x)^{21}$. Then $P(2016)$ equals to
 (A) 0, (B) 20^{2016} , (C) 2016, (D) 2015^{20} .
14. We define an operation $*$ as follows: $a * b = \frac{a-b}{1-ab}$. Then $1 * (2 * (3 * \dots (2015 * 2016))) \dots =$
 (A) $2016 \times 2015 \times \dots \times 1$, (B) $\frac{1}{1-2016 \times 2015}$, (C) $\frac{2016}{2015}$, (D) None of these.
15. Let n be a two digit number such that

$$\text{sum of digits of } n + \text{product of digits of } n = n$$

Then the unit digit of n is

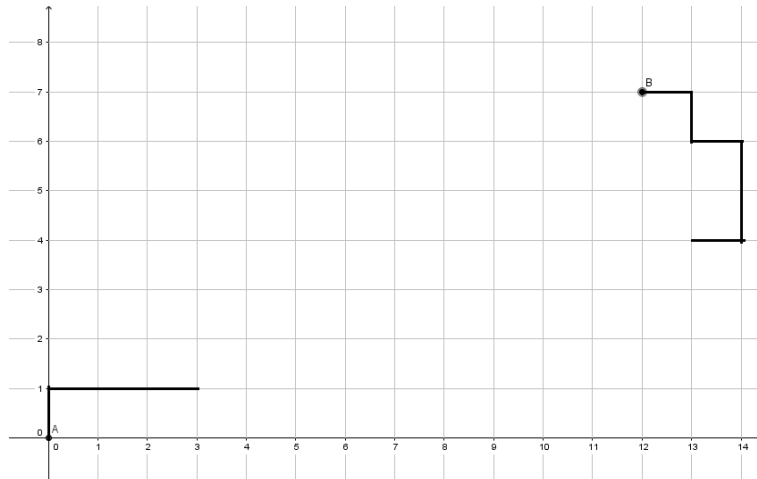
- (A) 1, (B) 9, (C) 7, (D) can't be determined.

Short Answer Type Questions

[Each question carries a total of 15 marks. Credit will be given to partially correct answers]

1. Let ABC be a triangle with $AB = AC$. The bisector of $\angle ACB$ meet AB at M . Suppose $AM + MC = BC$. Show that $\angle BAC = 100^\circ$

2. Two friends A and B are initially at points $(0,0)$ and $(12,7)$ respectively on the infinite grid plane (see figure). A takes steps of size 4 units and B takes steps of size 6 units along the grid lines. For example, a permissible step of A and B are shown in the figure [They are not necessarily the initial steps of A and B]. Show that it is not possible for them to meet at a point.



3. Mtrpia, a small country, has the following coins in circulation: 1 paise, 2 paise, 5 paise, 10 paise, 20 paise, 50 paise, and 1 rupee. Suppose it is known that you can pay A paise with B coins. Prove that you can pay B rupees with A coins. [Assume that there are infinitely many coins of each type.]
4. Consider the following positive integers

$$a, a + d, a + 2d, a + 3d, \dots$$

Suppose there is a perfect square in the above list of numbers. Then prove that there are infinitely many perfect squares in the above list.

5. 2016 coins are placed on a table with 50 heads up and remaining tails up. Suppose you are blindfolded and only thing you can do is flip the coins. Explain how you can separate the 2016 coins into two groups such that each group has equal number of heads.
6. Find all positive integers x and y such that $x, y, x + y$ and $x - y$ all are primes.

Use of calculators is not allowed. You may use a ruler and a compass for construction.
 ~ Best of Luck ~