Mathematics Talent Reward Programme

Model Solutions for Class IX

Multiple Choice Questions

[*Each question has only one correct option. You will be awarded 4 marks for the correct answer, 1 mark if the question is not attempted and 0 marks for wrong answer.*]

1. **(C)**	2. **(B)**	3. **(A)**	4. **(D)**	5. **(A)**
6. **(C)**	7. **(D)**	8. **(B)**	9. **(D)**	10. **(B)**
11. **(C)**	12. **(A)**	13. **(A)**	14. **(D)**	15. **(B)**

Short Answer Type Questions

[*Each question carries a total of 15 marks. Credit will be given to partially correct answers*]

1. Let D be a point on BC such that $CM = CD$. Then we have

$$AM + MC = BC = BD + CD = BD + CM \implies AM = BD$$

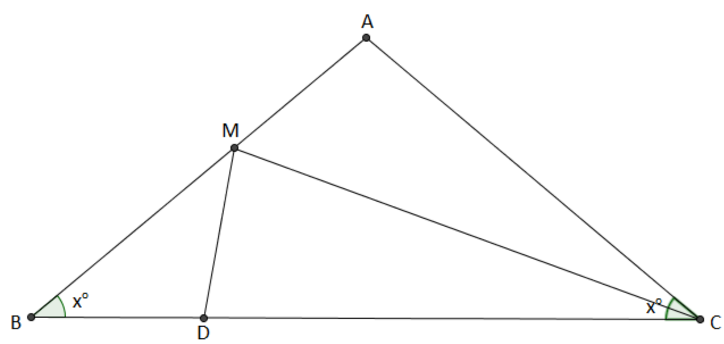
Now consider the triangles $\triangle BMD$ and $\triangle ABC$. We have $\angle MBD = \angle ACB$ and since CM is a bisector of $\angle ACB$, we have

$$\frac{AC}{BC} = \frac{AM}{BM} = \frac{BD}{BM}$$

Thus $\triangle BMD \sim \triangle ABC$. Let $\angle ABC = \angle ACB = x$ Then $\angle BMD = x$. Thus $\angle MDC = \angle BMD + \angle MBD = 2x$. Note that $CM = CD \implies \angle DMC = 2x$. Hence if we consider the angles of triangle $\triangle CMD$ we have

$$2x + 2x + \frac{x}{2} = 180^\circ \implies x = 40^\circ$$

This implies $\angle BAC = 180^\circ - 2 \times 40^\circ = 100^\circ$.



□

2. Consider the parity on the sum of the co-ordinates of postions of A and B separately and note that for each step (4 for A, 6 for B), the parity of the sum of the co-ordinates does not change. Hence A having sum of the co-ordinates 0 (even) initially and B having sum of the co-ordinates 19 (odd) initially can never meet. □

3. Let x_i be the number of coins of i-th type used for paying A paise. Then we have

$$x_1 + x_2 + \cdots + x_7 = B, \qquad x_1 + 2x_2 + 5x_3 + \cdots + 100x_7 = A$$

Observe that

$$\begin{aligned}100B &= 100x_1 + 100x_2 + \cdots + 100x_7 \\ &= 100 \times x_1 + 50 \times 2x_2 + 20 \times 5x_3 + \cdots 1 \times 100x_7\end{aligned}$$

Now if we define $y_1 = x_1, y_2 = 2x_2, y_3 = 5x_3, \ldots y_7 = 100x_7$ we have

$$y_1 + y_2 + \cdots + y_7 = A, \qquad 100y_1 + 50y_2 + 20y_3 + \cdots + y_7 = 100B$$

Thus if we use y_7 1 paisa coins, y_6 2 paise coins, y_5 5 paise coins, ... , y_1 1 rupee coins, we can pay B rupees using A coins. □

4. Suppose there is a square x^2 in that list. Observe that

$$(x + d)^2 = x^2 + 2xd + d^2 = x^2 + (2x + d)d$$

is of the form $x^2 + kd$ which must be in that list. Thus considering $x^2, (x + d)^2, (x + 2d)^2 \ldots$ we get a list of infinite perfect squares which is a sublist of the original list. □

5. Take any 50 coins from 2016 coins to form heap A. The remaining 1966 coins form heap B say. Suppose there are x coins in heap A with heads facing up and hence there are $50 - x$ coins in heap A with tails facing up. If we flip all the coins of heap A, then we will get $50 - x$ coins of A with heads facing up. Note that there are $50 - x$ coins in heap B with heads facing up. This completes the proof. □

6. Since $x - y$ is a prime, $x - y > 0 \implies x > y$. Suppose both $x, y \geq 3$, then $x + y$ becomes even and hence not a prime. So one of them must be 2. Hence $y = 2$ and $x \geq 3$. So we have $x - 2, x, x + 2$ as primes. Consider three cases:

   <u>Case 1</u>: $x = 3k + 1$ where $k \geq 1$, then $x + 2 = 3k + 3 = 3(k + 1)$ which is certainly not a prime.

   <u>Case 2</u>: $x = 3k + 2$ where $k \geq 1$, then $x - 2 = 3k$ which is prime only if $k = 1$. This forces $x = 5$. A simple checking shows that this is indeed a solution.

   <u>Case 3</u>: $x = 3k$ where $k \geq 1$, then $k = 1$, which forces $x - y = 1$, not a prime.

   Thus $x = 5, y = 2$ is the only solution. □